Dependable Dynamic Routing for Urban Transport Systems Through Integer Linear Programming

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Overline of the presentation

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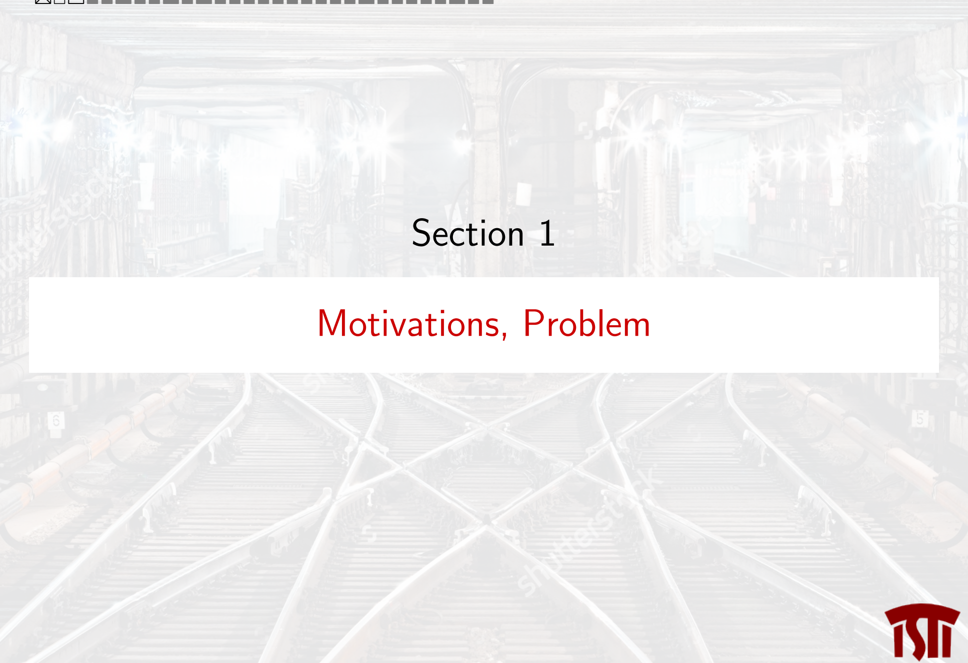


Section 1

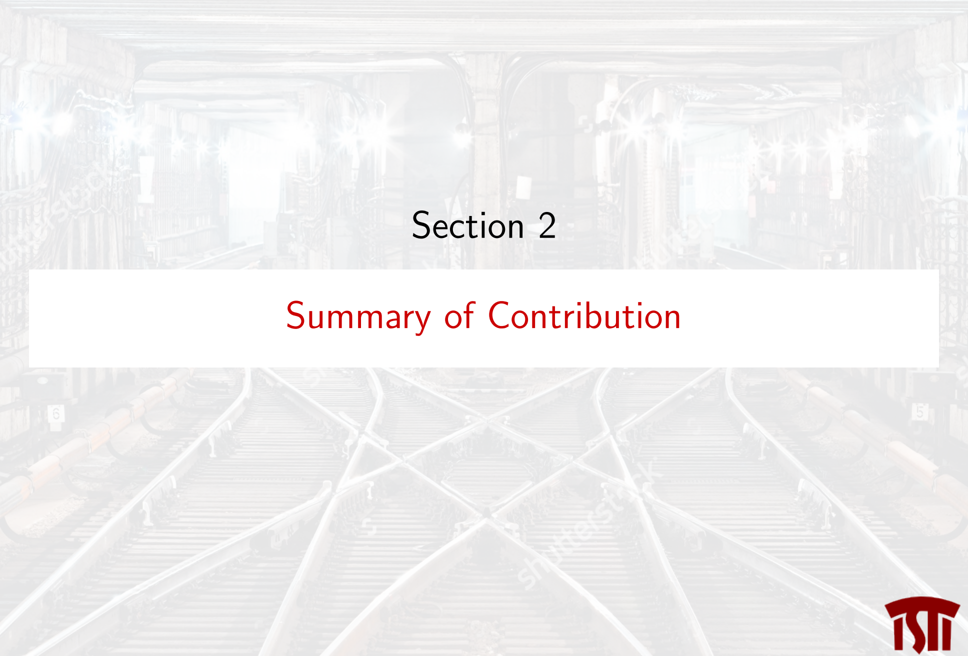
Motivations, Problem

Motivations

- Motivations: build novel solutions for transport systems, reduce *costs*, improve *sustainability*, *reliability* and *safety*;
- Driver-less transport solutions already deployed in metropolitan cities;
- **Optimal vehicles routes** → reduce costs, energy consumption, improve delivered service, user satisfiability;
- Traditionally two-step approaches: planning and execute;
    - assumption: track availability,
    - *problem*: not possible in urban area, e.g. track obstructions;
- *solution*: **Dynamic vehicles routing**, possible through novel technologies: GPS, GNSS, GIS, ITS, etc...

Dynamic Vehicle Routing Problem

- Widely studied and adopted for planning routes of vehicles and supervising their movements,
- Recently, solutions for train and subway lines have been extended to other urban transport systems, e.g. tramway lines,
- *Tramway lines*: characteristics
  - signals, priorities, traffic lights already adopted,
  - driver to enforce speed, braking, safety distance,
  - **problem**: tracks obstructions due to accidents, other vehicles etc..., needs to adopt temporary routes until normal operations are restored,
  - **innovation**: radar and gps systems to detected obstructions, dynamic routing to improve user satisfaction, energy consumption, etc...



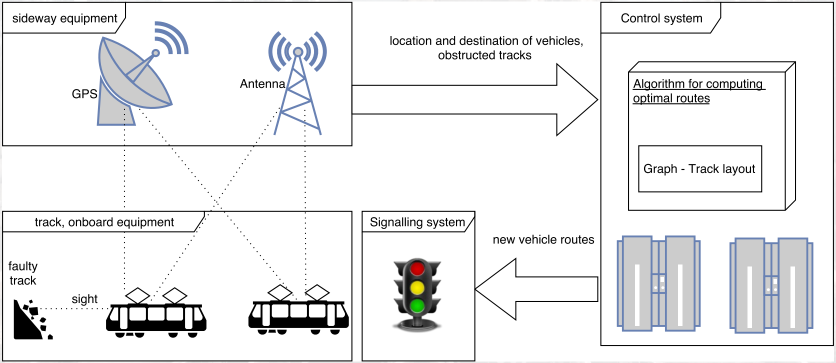
Section 2

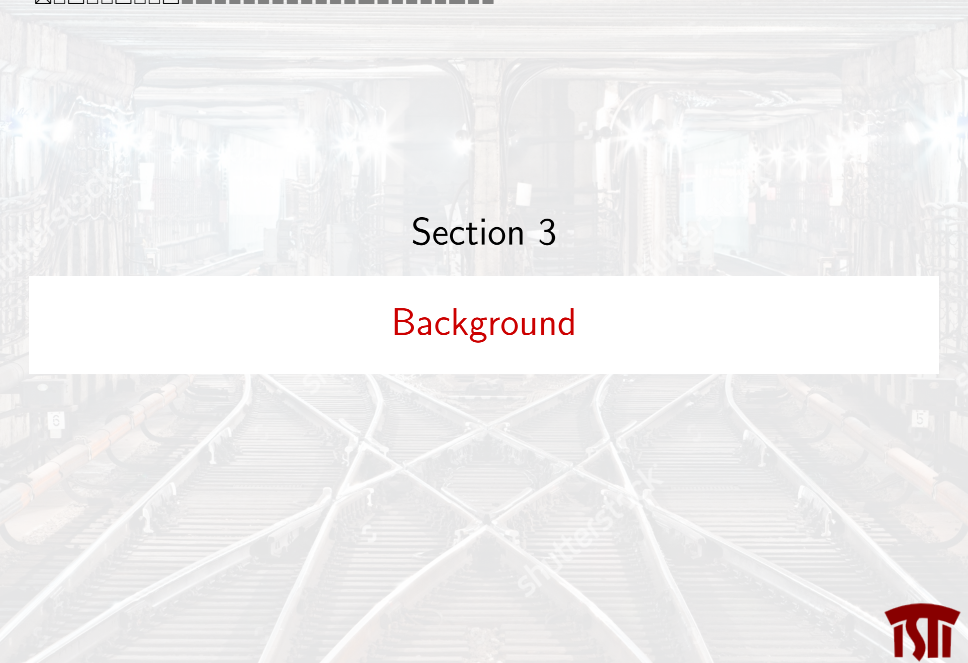
Summary of Contribution

Contribution

- **Dynamic routing algorithm** for urban area:
    - possible obstructed tracks can be detected,
    - new routes are dynamically recomputed
- Input: graph abstracting an urban map, location and destination of vehicles, unavailable tracks
    - nodes: points, edges: itineraries
- Output: **optimal routes** for each vehicle (deteriorated tracks) s.t.:
    - collisions on itineraries or points are *ruled out* on routes,
    - obstructed tracks are *not traversed* by routes,
    - *progress* of the network, *minimum* arriving time for all vehicles.
- modelled as an *optimization combinatorial problem*:
    - routes are flows in a graph (flow problem),
    - objective function: minimise arrival time,
    - linear constraints ensuring safety,
    - open-source implementation in AMPL.

Dynamic Vehicle Routing Architecture



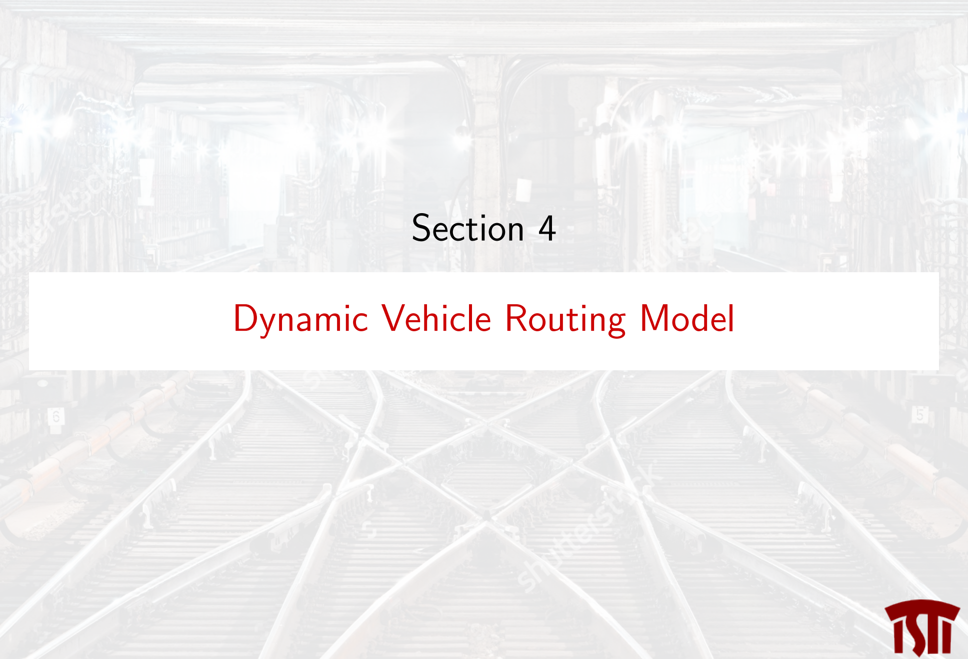


Section 3

Background

Network Flow Problem

- Flow network: directed graph $G = (Q, T)$ with source and sink, each edge has capacity and receives flow,
- $\forall q \in Q. FS(q)$ forwad star, $BS(q)$ backward star,
- $\forall t \in T. x_t$ flow variable, a_t capacity, c_t cost,
- network flow problem: optimize flow s.t.:
  - $\forall t \in T. x_t \leq a_t$ capacity,
  - $\forall q \in Q. \sum_{t \in BS(q)} x_t - \sum_{t \in FS(q)} x_t = \begin{cases} -d & \text{if } q = q_s \\ 0 & \text{if } q \neq q_s, q_f \\ d & \text{if } q = q_f \end{cases}$
  - $\forall t \in T. x_t \in \mathbb{N}$ integrity
- Objective function examples:
  - $\texttt{max}\ \ d$ (maximum flow problem)
  - $\texttt{min}\ \ \sum_{t \in T} x_t c_t$ (minimum cost flow problem)
- solved through ILP, e.g. simplex algorithm



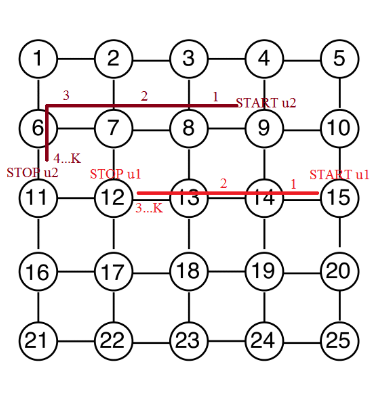
Section 4

Dynamic Vehicle Routing Model

Dynamic Vehicle Routing Model

- modeled as a *network flow problem*,
- routes will be flow in the graph abstracting the urban map:
  - maximum capacity $a_t = 1$ (only one vehicle in each itinerary),
  - flow $d = 1$ (each flow corresponds to a route),
  - costs left as future work (energy, performance, etc...)
- round-trips are not considered (e.g. destination: next-stop)
- U set of *vehicles*,
- it is possible to split the flow variables into time steps $1, \ldots, K$
  - vehicles can be in the same edge in different time steps,
- $K = |U| * |T|$: upper-bound to max. num. of steps,
- flow variables (routes) $x_{u,k,t}$:
  - $u \rightarrow$ vehicle, $k \rightarrow$ discrete time step, $t \rightarrow$ itinerary
- $x_{u,k,t} = 1$: vehicle u at time step k is in itinerary t,
- the model will optimize the values of variables $x_{u,k,t}$.

Dynamic Vehicle Routing Model (2)

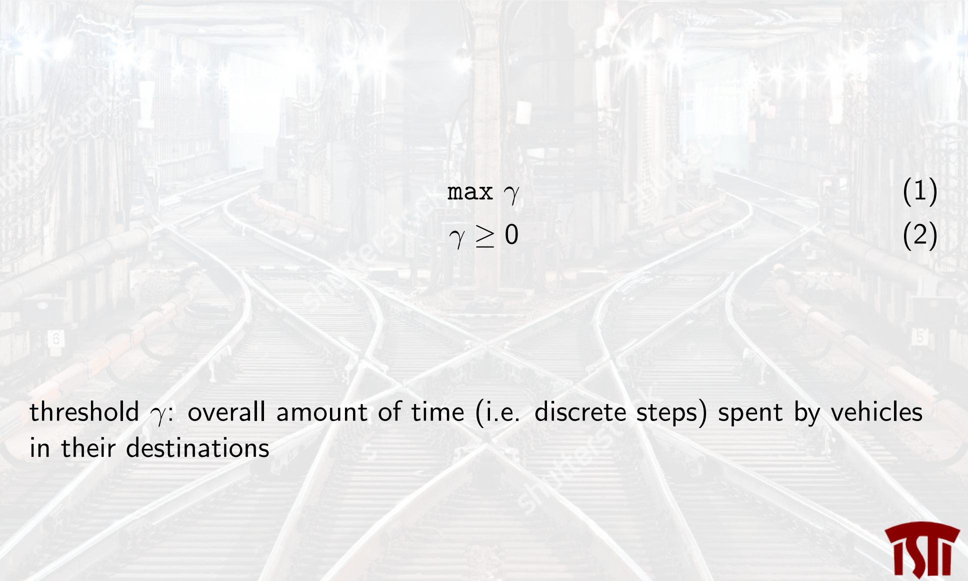


$$F = \{(1,2),(1,6),(2,1),(2,7),(6,1),(7,2),(10,2)\}$$

$$x_{u_1,1,(15,14)} = 1, x_{u_1,2,(14,13)} = 1, x_{u_1,i,(13,12)} = 1 \; i = 3,\ldots,K$$

$$x_{u_2,1,(9,8)} = 1, x_{u_2,2,(8,7)} = 1, x_{u_2,3,(7,6)} = 1, x_{u_2,j,(6,11)} = 1, \; j = 4,\ldots,K$$

Objective Function



$$\mathtt{max}\ \gamma \tag{1}$$

$$\gamma \geq 0 \tag{2}$$

threshold γ: overall amount of time (i.e. discrete steps) spent by vehicles in their destinations

Flow Constraints

binary flow variables

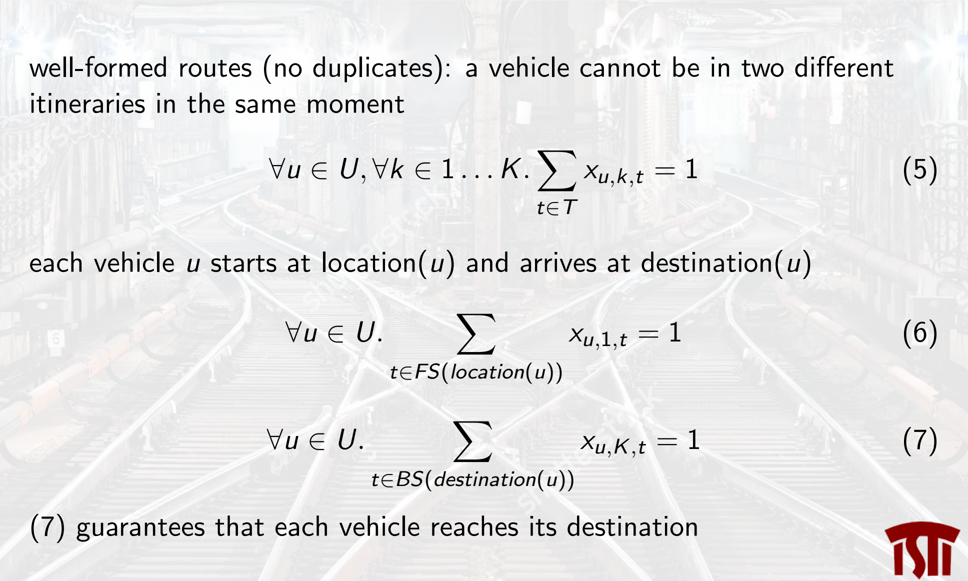
$$\forall k \in 1 \ldots K, \forall u \in U, \forall t \in T.\ x_{u,k,t} \in \{0,1\} \tag{3}$$

minimum arrival time:

- $\sum_{u \in U, k \in K, t \in BS(destination(u))} x_{u,k,t}$: amount of time that vehicles spent in their destinations
- at most $|K| * |U|$: the problem is bounded
- by maximizing this value the length of each route is minimized: the later vehicles arrive, the small the sum will be

$$\sum_{u \in U, k \in K, t \in BS(destination(u))} x_{u,k,t} \geq \gamma \tag{4}$$

Flow Constraints (1)



well-formed routes (no duplicates): a vehicle cannot be in two different itineraries in the same moment

$$\forall u \in U, \forall k \in 1 \ldots K. \sum_{t \in T} x_{u,k,t} = 1 \tag{5}$$

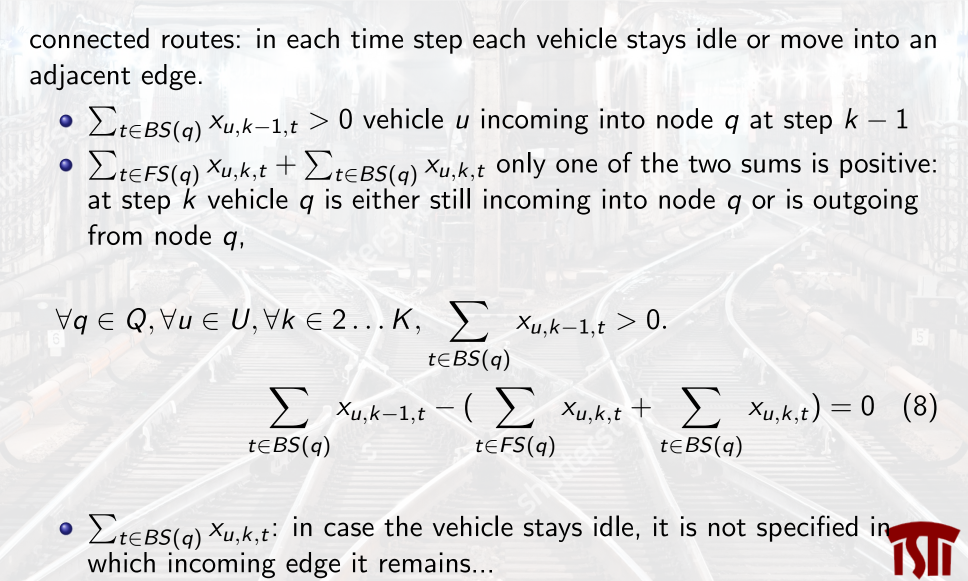
each vehicle u starts at location(u) and arrives at destination(u)

$$\forall u \in U. \sum_{t \in FS(location(u))} x_{u,1,t} = 1 \tag{6}$$

$$\forall u \in U. \sum_{t \in BS(destination(u))} x_{u,K,t} = 1 \tag{7}$$

(7) guarantees that each vehicle reaches its destination

Flow Constraints (2)



connected routes: in each time step each vehicle stays idle or move into an adjacent edge.

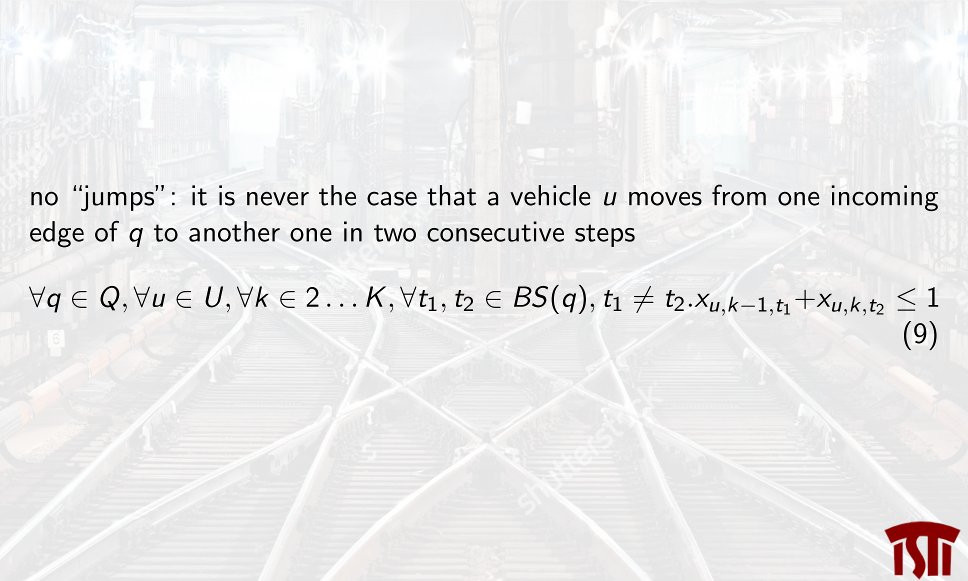
- $\sum_{t\in BS(q)} x_{u,k-1,t} > 0$ vehicle u incoming into node q at step $k-1$
- $\sum_{t\in FS(q)} x_{u,k,t} + \sum_{t\in BS(q)} x_{u,k,t}$ only one of the two sums is positive: at step k vehicle q is either still incoming into node q or is outgoing from node q,

$$\forall q \in Q, \forall u \in U, \forall k \in 2 \dots K, \sum_{t\in BS(q)} x_{u,k-1,t} > 0.$$

$$\sum_{t\in BS(q)} x_{u,k-1,t} - (\sum_{t\in FS(q)} x_{u,k,t} + \sum_{t\in BS(q)} x_{u,k,t}) = 0 \tag{8}$$

- $\sum_{t\in BS(q)} x_{u,k,t}$: in case the vehicle stays idle, it is not specified in which incoming edge it remains...

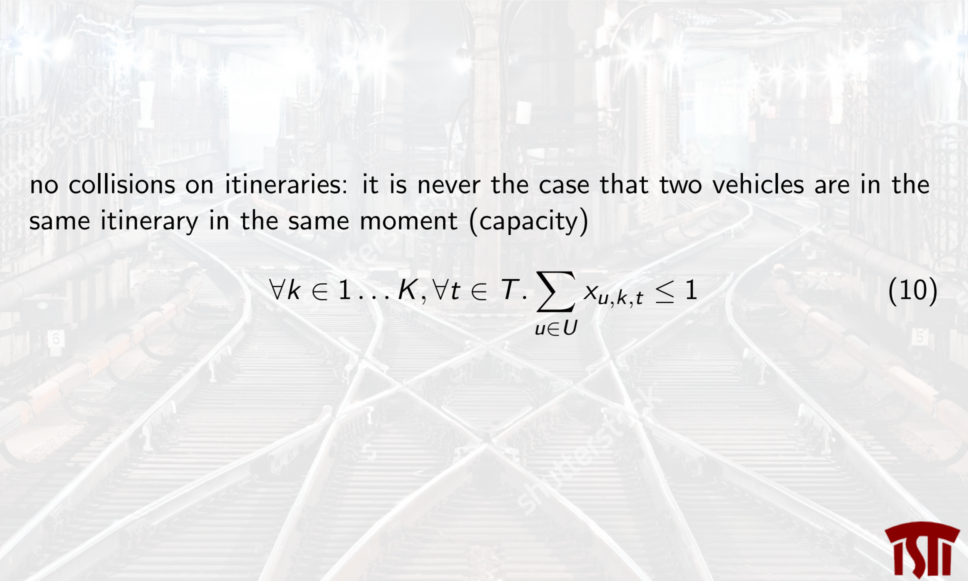
Flow Constraints (3)



no “jumps”: it is never the case that a vehicle u moves from one incoming edge of q to another one in two consecutive steps

$$\forall q \in Q, \forall u \in U, \forall k \in 2 \ldots K, \forall t_1, t_2 \in BS(q), t_1 \neq t_2 . x_{u,k-1,t_1} + x_{u,k,t_2} \leq 1 \tag{9}$$

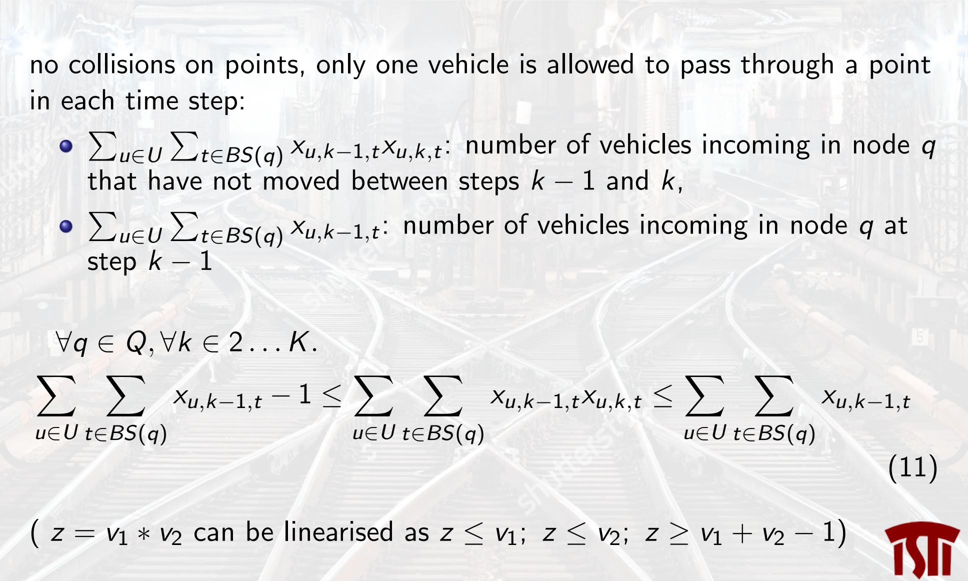
Safety Constraints



no collisions on itineraries: it is never the case that two vehicles are in the same itinerary in the same moment (capacity)

$$\forall k \in 1 \ldots K, \forall t \in T. \sum_{u \in U} x_{u,k,t} \leq 1 \tag{10}$$

Safety Constraints



no collisions on points, only one vehicle is allowed to pass through a point in each time step:

- $\sum_{u \in U} \sum_{t \in BS(q)} x_{u,k-1,t} x_{u,k,t}$: number of vehicles incoming in node q that have not moved between steps $k-1$ and k,
- $\sum_{u \in U} \sum_{t \in BS(q)} x_{u,k-1,t}$: number of vehicles incoming in node q at step $k-1$

$$\forall q \in Q, \forall k \in 2 \ldots K.$$

$$\sum_{u \in U} \sum_{t \in BS(q)} x_{u,k-1,t} - 1 \leq \sum_{u \in U} \sum_{t \in BS(q)} x_{u,k-1,t} x_{u,k,t} \leq \sum_{u \in U} \sum_{t \in BS(q)} x_{u,k-1,t} \tag{11}$$

($z = v_1 * v_2$ can be linearised as $z \leq v_1$; $z \leq v_2$; $z \geq v_1 + v_2 - 1$)

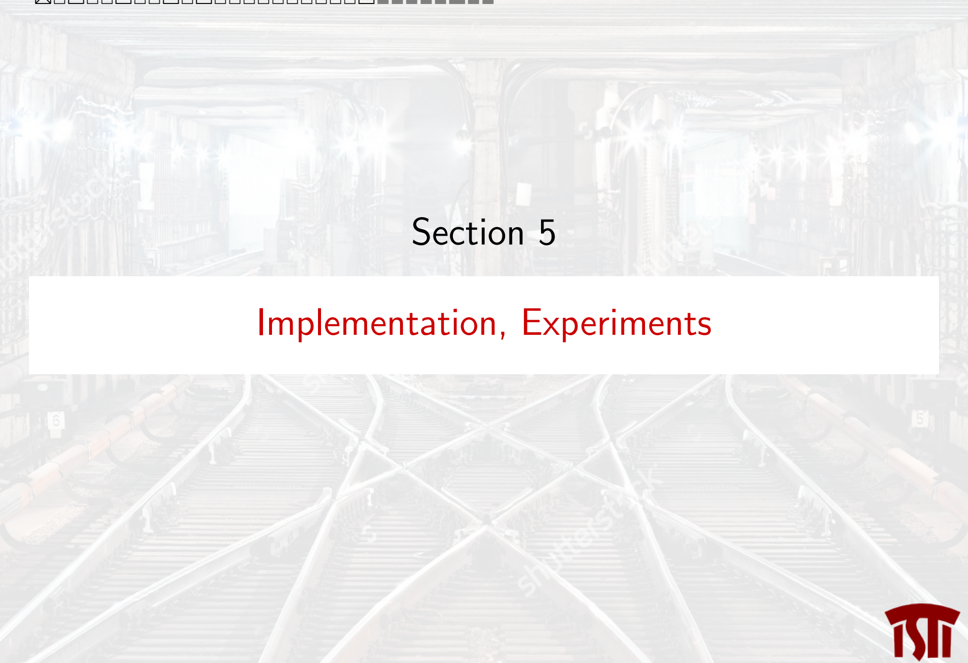
Safety Constraints - Graph Structure

unavailable itineraries are never traversed

$$\forall t \in F. \sum_{u \in U} \sum_{k \in 1\ldots K} x_{u,k,t} = 0 \tag{12}$$

graph structure: no inner cycles

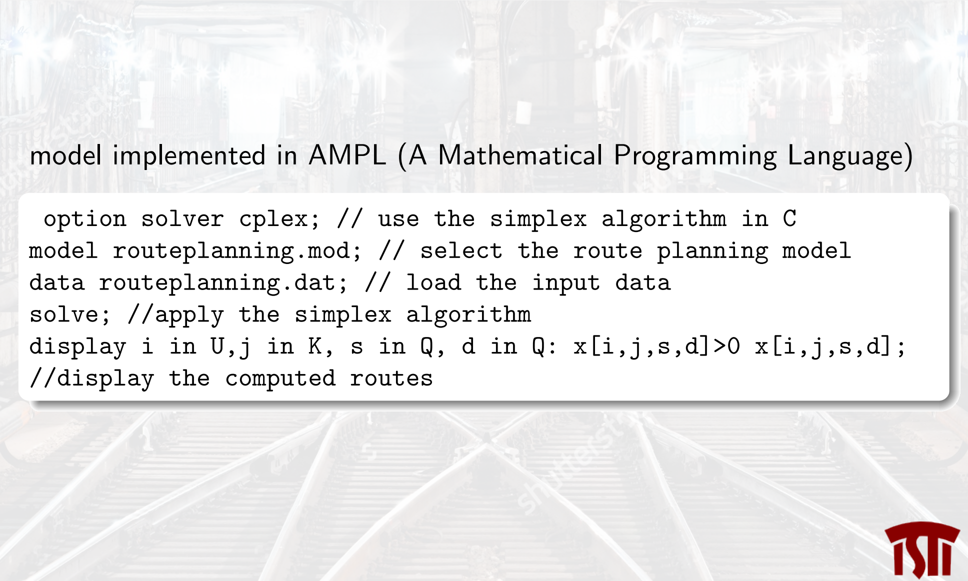
$$\forall q \in Q, \forall u \in U, \forall k \in 1 \ldots K, \forall t \in T. \sum_{t \in FS(q)} x_{u,k,t} + \sum_{t \in BS(q)} x_{u,k,t} \leq 1 \tag{13}$$



Section 5

Implementation, Experiments

Implementation



model implemented in AMPL (A Mathematical Programming Language)

```
 option solver cplex; // use the simplex algorithm in C
model routeplanning.mod; // select the route planning model
data routeplanning.dat; // load the input data
solve; //apply the simplex algorithm
display i in U,j in K, s in Q, d in Q: x[i,j,s,d]>0 x[i,j,s,d];
//display the computed routes
```

Implementation (2)

The input file `routeplanning.data` contains:

- number of vehicles u and nodes n,
- two binary matrix t and F
  - `t[q1,q2]=1` iff there is an edge $(q1, q2)$
  - `F[q1,q2]=1` iff itinerary $(q1, q2)$ is unavailable
- two arrays `location[u]` and `destination[u]`

Implementation (3)

The input file `routeplanning.mod` contains the model and additional constraints:
each flow only uses edges:

$$\forall u \in U, k \in K, s \in Q, d \in Q : x[u,k,s,d] <= t[s,d]$$

connected path:

$$\forall q \in Q, \forall u \in U, \forall k \in 2 \ldots K.$$

$$\sum_{t \in BS(q)} x_{u,k-1,t} - \Big(\sum_{t' \in FS(q), t \in BS(q)} x_{u,k,t'} \mathbf{x_{u,k-1,t}} + \sum_{t \in BS(q)} x_{u,k,t} \mathbf{x_{u,k-1,t}} \Big) = 0$$

linearization constraints

Experiments

random generation of obstructed tracks, locations and destinations

	Experiment 1
Obstructed tracks	(2,7) (6,7) (9,8) (11,6) (13,14) (17,12) (19,18) (22,23) (23,24)

Vehicle 1	location=1, destination=25
	route=(1,2)(2,3)(3,8)(8,9)(9,14)(14,15)(15,20)(20,25)

Vehicle 2	location=25, destination=1
	route=(25,20)(20,15)(15,10)(10,5)(5,4)(4,3)(3,2)(2,1)

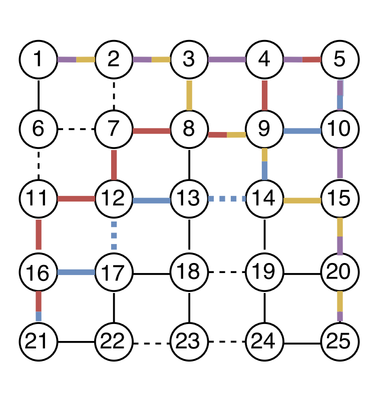
Vehicle 3	location=5, destination= 21
	route=(5,10)(10,9)(9,14)(14,13)(13,12)(12,17)(17,16)(16,21)

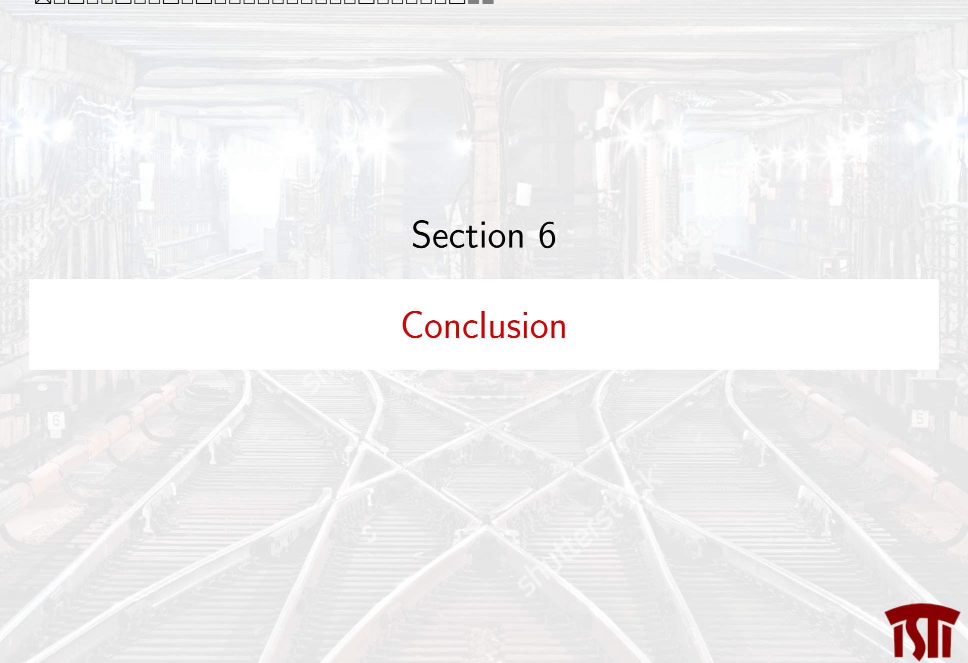
Vehicle 4	location=21, destination=5
	route=(21,16)(16,11)(11,12)(12,7)(7,8)(8,9)(9,4)(4,5)

	Experiment 2
Obstructed tracks	(1,6) (2,1) (2,7) (6,1) (6,7) (7,2) (9,8) (10,2) (10,15) (23,22)
Vehicle 1	location=14, destination=11, route=(14,13)(13,12)(12,11)
Vehicle 2	location=8, destination=10, route=(8,9)(9,10)
Vehicle 3	location=13, destination=18, route=(13,18)
Vehicle 4	location=21, destination=12, route=(21,16)(16,11)(11,12)
	Experiment 3
Obstructed tracks	(1,2) (1,6) (2,1) (2,7) (6,1) (7,2) (10,2)
Vehicle 1	location=15, destination=12, route=(15,14)(14,13)(13,12)
Vehicle 2	location=9, destination=11, route=(9,8)(8,7)(7,6)(6,11)
Vehicle 3	location=14, destination=19, route=(14,19)
Vehicle 4	location=22, destination=13, route=(22,17)(17,12)(12,13)

Experiment (2)

Experiment 1:





Section 6

Conclusion

Conclusion

- Dependable dynamic vehicle routing through ILP:
  - input: urban map (abstracted as a graph), vehicles locations and destinations (e.g. next stop), obstructed tracks,
  - output: set of routes with guarantees on,
    - absence of deadlocks, absence of collisions, minimum arriving time:
  - modeled as a flow problem on graph (nodes are points, edges are itineraries)
  - implemented in AMPL
- Future work:
  - apply to a (portion of a) real world urban scenario
  - include performance and energy aspects (e.g. speed, braking, fuel)
  - simulations on a whole day with failures

thanks for your attention